

Some useful quantities you may wish to calculate to assess the accuracy of the WKB approximation, its “trapezoidal estimate” and your own calculation from Lab 2:

(a) the absolute error of the WKB approximation:

$$\delta E_n^{\text{WKB}} = E_n^{\text{WKB}} - E_n, \quad (1)$$

where E_n is the exact energy of level n (obtained in this lab) and E_n^{WKB} is the corresponding energy in the WKB approximation (column 2 of `WKB_levels.dat`);

(b) the error of the WKB approximation relative to the exact energy of the level:

$$\epsilon_n^{\text{WKB}} = \frac{|\delta E_n^{\text{WKB}}|}{E_n}; \quad (2)$$

(c) the error of the WKB approximation relative to the distance between the levels:

$$\gamma_n^{\text{WKB}} = \frac{|\delta E_n^{\text{WKB}}|}{E_{n+1} - E_n}; \quad (3)$$

(d) the error of the WKB approximation relative to the difference between the energy level of the anharmonic oscillator and the corresponding level E_n^{harm} of the harmonic oscillator ($m = 2$, $\alpha = 1$, $\beta = 0$, $\hbar = 0.1$) with the same n :

$$\eta_n^{\text{WKB}} = \frac{|\delta E_n^{\text{WKB}}|}{|E_n - E_n^{\text{harm}}|}; \quad (4)$$

(e) the same four quantities using the “trapezoidal estimate” of the WKB approximation, i.e., E_n^{trap} from column 3 of `WKB_levels.dat`:

$$\delta E_n^{\text{trap}} = E_n^{\text{trap}} - E_n, \quad (5)$$

$$\epsilon_n^{\text{trap}} = \frac{|\delta E_n^{\text{trap}}|}{E_n}; \quad (6)$$

$$\gamma_n^{\text{trap}} = \frac{|\delta E_n^{\text{trap}}|}{E_{n+1} - E_n}; \quad (7)$$

$$\eta_n^{\text{trap}} = \frac{|\delta E_n^{\text{trap}}|}{|E_n - E_n^{\text{harm}}|}. \quad (8)$$

(f) the same four quantities using your own approximation E_n^{my} to the WKB approximation (if you have done it):

$$\delta E_n^{\text{my}} = E_n^{\text{my}} - E_n, \quad (9)$$

$$\epsilon_n^{\text{my}} = \frac{|\delta E_n^{\text{my}}|}{E_n}; \quad (10)$$

$$\gamma_n^{\text{my}} = \frac{|\delta E_n^{\text{my}}|}{E_{n+1} - E_n}; \quad (11)$$

$$\eta_n^{\text{my}} = \frac{|\delta E_n^{\text{my}}|}{|E_n - E_n^{\text{harm}}|}. \quad (12)$$

(g) the error of the “trapezoidal estimate” of the WKB approximation compared to the “exact” WKB result:

$$\Delta E_n^{\text{trap}} = E_n^{\text{trap}} - E_n^{\text{WKB}}; \quad (13)$$

(h) the error of the “trapezoidal estimate” of the WKB approximation relative to the error of the WKB approximation itself:

$$\xi_n^{\text{trap}} = \frac{|\Delta E_n^{\text{trap}}|}{|\delta E_n^{\text{WKB}}|}; \quad (14)$$

(i) same as in (g) and (h) for your own data, if you have them:

$$\Delta E_n^{\text{my}} = E_n^{\text{my}} - E_n^{\text{WKB}}; \quad (15)$$

$$\xi_n^{\text{my}} = \frac{|\Delta E_n^{\text{my}}|}{|\delta E_n^{\text{WKB}}|}. \quad (16)$$