Project 1

Due Friday, October 25, 2013

Note that some of the problems are somewhat open-ended. Those who do more than roughly 75% of the students will get more than 100% of the points provided that they do everything they chose to do perfectly.

1. **(Optional for PHY4140 students)** (3 pts.) Write a program doing a Fast Fourier Transform (FFT) on a data set with N samples. You do not have to allow a general N: $N = 2^n$, where n is an integer, is enough. Generate a few test data sets and test your code. **Use your FFT code in at least one of the following tasks.** You can apply it to any data set generated by any of the tasks, but the more "natural" this application feels and the more it helps you solve the problem, the better.

2. (10 pts.) Consider the following dynamical system:

$$\begin{aligned} \dot{x} &= -y - z, \\ \dot{y} &= x + ay, \\ \dot{z} &= b + z(x - c), \end{aligned} \tag{1}$$

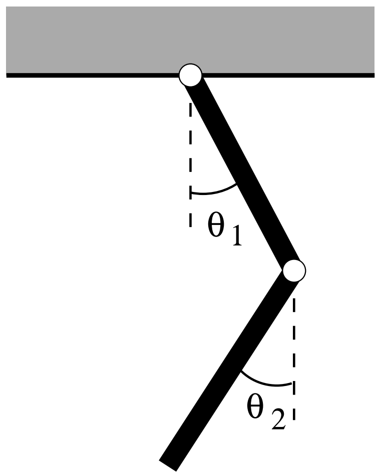
where a, b and c are constant parameters. Study this system in as much detail as you find appropriate. Describe your study in detail. Provide all codes. You do not need to submit all data sets. At the very minimum, do the following:

(a) consider $b = 2$ and $c = 4$ and vary a. Find the fixed points [values of (x, y, z) for which $\dot{x} = \dot{y} = \dot{z} = 0$] analytically and do the stability analysis for these fixed points either analytically or combining analytical and numerical calculations. Study the system numerically for different a (you can re-use and adapt the code you've used in the previous lab if you prefer). In particular, study the character of the steady-state motion of the system. At what values of a does the character of the motion change qualitatively (bifurcations)? Try to find accurate positions of as many bifurcations as you can. What are the types of these bifurcations? Do you observe chaos for any a? How does the transition to chaos occur?

(b) study the chaotic regime. At the very least, plot a projection of the attractor for one value of a and do a Poincaré section. Consider studying the fractality of the attractor and the Lyapunov exponents.

You do not need to limit yourself to the $b = 2$, $c = 4$ case only.

3. (7 pts.) Consider a frictionless double pendulum (see the figure). Write down the equations of motion for the angles θ_1 and θ_2 assuming that the arms are uniform rigid rods and their lengths are l_1 and l_2. Assume that angles θ_1 and θ_2 can take any values (i.e., they are not limited to, e.g., the interval from $-\pi/2$ to $\pi/2$). For the case $l_1 = l_2 = l$ study numerically the behaviour of the system for different energies and initial conditions. Keep in mind that this is a conservative system, so using a symplectic method for solving the ODEs may be appropriate (if you don't, justify not using it by checking if the energy is conserved to a sufficiently high precision during your calculations). Plot some Poincaré sections for different energies to illustrate how the behaviour of the system changes as the energy increases. For each Poincaré section, fix the energy and plot the results for different initial conditions consistent with that energy. For up to **3 bonus points**, you can also consider the case $l_1 \neq l_2$.



4. **Optional for PHY4140 students** (5 pts.) Consider the following differential equation:

$$\ddot{x} = \mu(1 - x^2)\dot{x} - x. \tag{2}$$

Solve this equation for $\mu = 0.1$, 1, 2, 5, 10, 20, 50 and 100 (and also 200, 500, and 1000 for **up to 2 bonus points**). For **at least five** of the above values of μ (choose the most representative ones in your view), use **both** a simple explicit fixed-time-step method (e.g., 4th-order Runge-Kutta) and at least one of the following: an implicit method, an adaptive time-step method or (**for 2 bonus points**) a method that is both implicit and has an adaptive time step. For the rest of the values of μ use a method that you find most appropriate. What is the character of the steady-state motion in each case? If you observe oscillations, find their period with the accuracy of 0.1%. For those cases where you have used two methods, use both methods to find the period and compare the result and the efficiency of the methods. Plot the phase portrait of the system (i.e., the trajectories of the steady-state motion in the $(x, \dot{x})$ plane) for each μ (choose one of the methods for each μ) combining the results for different μ in one plot (or two plots, one for smaller μ and one for larger μ, if you find that the scales differ too much). Describe the phase portrait and how it evolves with μ qualitatively. Also, for $\mu = 5$ plot the phase portrait including the full evolution of the system to the steady state starting with different initial conditions.