WKB approximation results in `WKB_levels.dat`

In the last question of Problem 2 of Lab 2, the energy levels were calculated "exactly" (of course, still within the WKB approximation) by calculating

$$S(E) = 2\int_{-A}^{A} |p(x)|dx \tag{1}$$

directly by numerical integration and using a root-finding procedure to solve the equation for the energy levels E_n:

$$S(E_n) = 2\pi\hbar(n + 1/2). \tag{2}$$

The results obtained using this approach for $\alpha = 1$, $\beta = 1$, $m = 2$, $\hbar = 0.1$ are given in the second column of `WKB_levels.dat` (the first column is the level numbers $0, 1, 2, \ldots$).

Another, more indirect and therefore potentially more approximate way was based on calculating $S(E)$ at a discrete set of points as

$$S(E) = \int_{0}^{E} T(E)dE \tag{3}$$

using the results for $T(A)$ obtained in Problem 1 of that lab and then interpolating between those points to solve Eq. (2). One possible way of doing this is using the trapezoidal rule for $S(E)$. Given a set of data for $T(A^{(i)})$, as obtained in Problem 1, where $A^{(i)} = 0.01i$ and $i = 0, 1, \ldots$,

$$S(E(A^{(i)})) \approx \frac{1}{2}\sum_{j=1}^{i}[T(A^{(j-1)}) + T(A^{(j)})][E(A^{(j)}) - E(A^{(j-1)})], \tag{4}$$

where

$$E(A^{(i)}) = \alpha(A^{(i)})^2 + \beta(A^{(i)})^4. \tag{5}$$

Then linear interpolation was used and Eq. (2) was solved. The results obtained using this approach for $\alpha = 1$, $\beta = 1$, $m = 2$, $\hbar = 0.1$ are given in the third column of `WKB_levels.dat`.